

2/28/18

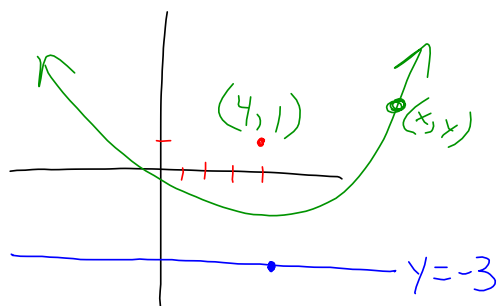
"Its easy to be happy when people do what they're supposed to."- Chris Callahan

HW: "Direct Variation" Homework section Test 2 on Friday 3/9

AIM: What is Direct Variation?

Warm Up:

Determine the equation of the parabola whose focus is the point $(4, 1)$ and whose directrix is the horizontal line $y = -3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.



Distance from Directrix = Distance to Focus

$$y - (-3) =$$

$$y + 3 = \sqrt{(x - 4)^2 + (y - 1)^2}$$

$$(y + 3)^2 = \left(\sqrt{x^2 - 8x + 16 + y^2 - 2y + 1} \right)^2$$

$$\begin{array}{rcl} y^2 + 6y + 9 & = & x^2 - 8x + 16 + y^2 - 2y + 1 \\ -y^2 + 2y & & -y^2 + 2y \end{array}$$

$$\begin{array}{rcl} 8y + 9 & = & x^2 - 8x + 17 \\ -9 & & -9 \end{array}$$

$$\frac{8y}{8} = \frac{x^2}{8} - \frac{8x}{8} + \frac{8}{8}$$

$$y = \frac{1}{8}x^2 - x + 1$$

PROPORTIONAL OR DIRECT RELATIONSHIPS

Two variables, x and y , have a **direct (proportional) relationship** if for every ordered pair (x, y) we have:

$$\frac{y}{x} = k \text{ or } y = kx$$

Stated succinctly, y will always be a constant multiple of x . The value of k is known as the **constant of variation**.

constant of variation

Exercise #1: In each of the following, x and y are directly related. Solve for the missing value.

(a) $y = 15$ when $x = 5$

$y = ?$ when $x = 9$

~~$$\frac{y}{x} = \frac{15}{5} = \frac{y}{9}$$~~

$$15(9) = 5y$$

$$\frac{135}{5} = \frac{5y}{5}$$

$$27 = y$$

(b) $y = -6$ when $x = 4$

$y = ?$ when $x = -10$

$$\frac{-6}{4} = \frac{y}{-10}$$

$$\frac{60}{4} = \frac{4y}{4}$$

$$y = 15$$

(c) $y = 12$ when $x = 16$

$y = ?$ when $x = 24$

$$\frac{16}{12} = \frac{24}{y}$$

$$\frac{288}{16} = \frac{16y}{16}$$

$$y = 18$$

$$\frac{12}{16} = \frac{y}{24}$$

$$288 = 16y$$

$$y = 18$$

Exercise #2: The distance a person can travel varies directly with the time they have been traveling if going at a constant speed. If Phoenix traveled 78 miles in 1.5 hours while going at a constant speed, how far will he travel in 2 hours at the same speed?

miles
hour

$$\frac{78}{1.5} = \frac{\text{miles}}{2}$$

$$104 = \text{miles}$$

$$\frac{156}{1.5} = \frac{1.5(\text{miles})}{1.5}$$

Exercise #3: Jenna works a job where her pay varies directly with the number of hours she has worked. In one week, she worked 35 hours and made \$274.75. How many hours would she need to work in order to earn \$337.55?

Pay

Pay
hours

$$\frac{274.75}{35} = \frac{337.55}{x}$$

Hours

$$\frac{274.75x}{274.75} = \frac{11814.25}{274.75}$$

$$x = 43 \text{ hours}$$

Exercise #4: Two variables, x and y , vary directly. When $x = 6$ then $y = 4$. The point is shown plotted below.
 (a) Find the y -values for each of the following x -values. Plot each point and connect.

$x = 3 \quad y = 2$

$$\frac{x}{y} = \frac{6}{4} = \frac{3}{y}$$

$$6y = 12$$

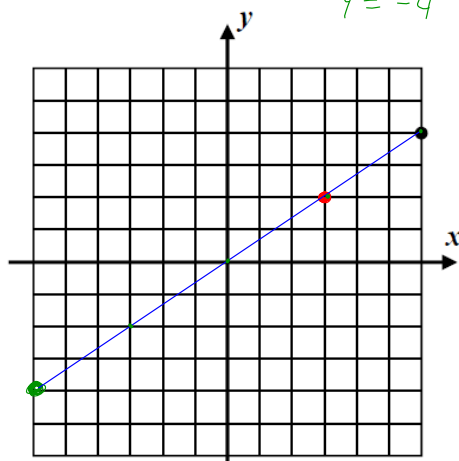
$$y = 2$$

$x = -6 \quad y = -4$

$$\frac{x}{y} = \frac{6}{4} = \frac{-6}{y}$$

$$6y = -24$$

$$y = -4$$



(b) What is the constant of variation in this problem? What does it represent on this line?

$$\frac{y}{x} = k \quad \frac{4}{6} = \frac{2}{3} = \frac{-4}{-6} = \boxed{\frac{2}{3}} \quad \text{Slope of line}$$

(c) Write the equation of the line you plotted in (a).

$y = mx + b$
 (slope intercept form)
 m is slope
 b is y -int.

$$y = \frac{2}{3}x + 0$$

$$y = \frac{2}{3}x$$

$y - y_1 = m(x - x_1)$
 (Point/slope form)
 (x_1, y_1) is a point
 m is slope

$(6, 4)$ is point

$$y - 4 = \frac{2}{3}(x - 6)$$

$$\begin{array}{r} y - 4 = \frac{2}{3}x - 4 \\ +4 \qquad \qquad +4 \\ \hline y = \frac{2}{3}x \end{array}$$

Direct relationships often exist between two variables whose values are zero simultaneously.

Exercise #3: The miles driven by a car, d , varies directly with the number of gallons, g , of gasoline used. Abigail is able to drive $d = 336$ miles on $g = 8$ gallons of gasoline in her hybrid vehicle.

- (a) Calculate the constant of variation for the relationship $\frac{d}{g}$. Include proper units in your answer.

$$\frac{336 \text{ miles}}{8 \text{ gallons}} = 42 \frac{\text{miles}}{\text{gallon}}$$

- (b) Give a linear equation that represents the relationship between d and g . Express your answer as an equation solved for d .

Slope/Intercept Let $y = d$
 $x = g$

$$y = 42x + 0$$

Point/Slope
 $(8, 336)$

$$y - 336 = 42(x - 8)$$

$$y - 336 = 42x - 336$$

$$y = 42x$$

$d = 42g \longleftrightarrow d = 42g$

- (c) How far can Abigail drive on $g = 6$ gallons of gas?

$$d = 42g$$

$$d = 42(6)$$

$$d = 252 \text{ miles}$$

$$\frac{42 \text{ miles}}{\cancel{\text{gallons}}} \cdot \frac{6 \cancel{\text{gallons}}}{1}$$

- (d) How many gallons of gas will Abigail need in order to drive 483 miles?

$$d = 42g$$

$$\frac{483}{42} = \frac{42g}{42}$$

$$11.5 \text{ gallons}$$